

**SATURN IB/S-IVB STAGE
SEPARATION CONTROLLABILITY REPORT**

(U)

**NOVEMBER 1964
(SUPERSEDES SM-44138)
DOUGLAS REPORT SM-46758**

**MISSILE & SPACE SYSTEMS DIVISION
DOUGLAS AIRCRAFT COMPANY, INC.
SANTA MONICA/CALIFORNIA**



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PREPARED FOR:
NATIONAL AERONAUTICS AND
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UNDER NASA CONTRACT NAS7-101



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PREFACE

The purpose of this report is to present the results of the Saturn IB separation control analysis for the revised 200,000 lb thrust H-1 engine and to update the previous separation report (reference 1) using the latest aerodynamic parameters. The effects of a 50 and 100 per cent dispersion in the aerodynamic parameters are included, as requested in Item 4.4 of the action items of the S-IVB Vehicle Dynamics and Control Working Group Meeting reported in MSFC Technical Direction No. I-V-S-IVB-TD-64-12 dated March 17, 1964 (reference 2).

This report is provided to partially fulfill the requirements of Contract Number NAS-7-101 as noted in Douglas Aircraft Company Report SM-41410; Data Submittal Document Saturn S-IVB System, Item 3.8, dated March 1962.

ABSTRACT

This report, using the latest aerodynamic, engine, and stage sequencing data available, investigates the controllability of the Saturn S-IVB/IB stage during separation from the S-IB stage. Post separation attitude transients and engine deflection transients are presented for a nominal condition using parameters which are slightly higher than the 90 per cent confidence level, 3σ values obtained from Saturn S-I flight data. Changes in these transient responses are presented for individual and combined dispersions of these parameters. The results indicate the control of the S-IVB stage can be maintained under nominal conditions and the system is stable. Maximum attitudes are produced by dispersions in the aerodynamic pressure. A 100 per cent dispersion in this parameter produced a 30° attitude excursion. Attitude excursions of 50° and 96° are obtained when this case is combined with the minimum guaranteed thrust rise curve or a positive 2 degree thrust misalignment, respectively. Further analyses to establish probabilities for these worse case excursions are required to determine a limit for the attitude rate which can be used in the emergency detection system.

DESCRIPTORS

S-IVB/IB

SEPARATION

CONTROLLABILITY

ATTITUDE

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LIST OF SYMBOLS, DEFINITIONS AND VALUES

<u>SYMBOL</u>	<u>DEFINITION</u>	<u>VALUE</u>
X	Initial Control Lever Arm	7.581 m
M	Initial Vehicle Mass	133,165 Kg
I	Initial Vehicle Inertia	8,259,819 Kg-m ²
ζ	Auto pilot damping ratio	0.7
A ₀	Position Loop gain (K_{θ}/K_{FB})	0.73 deg/deg
A ₁	Rate Loop gain ($K_{\dot{\theta}}/K_{FB}$)	0.9 deg/deg/sec
K _M	Forward Motor Loop Gain	2.15 deg/sec/ma
K _{FB}	Thrust Vector Control Feedback gain	6.64 ma/deg
A	Aerodynamic Reference Area	33.3 m ²
t _s	Separation Time	(See table 1)
q	Dynamic Pressure	(See table 2)
T	Main Engine Thrust	(See chart 1)
C _N	Aerodynamic Normal Force Coefficient	(See chart 3)
l _{CP}	Aerodynamic Lever Arm	(See chart 4)

VARIABLE PARAMETERS

θ	Attitude Angle	Degrees
$\dot{\theta}$	Attitude Rate	Deg/sec
t	Time	Sec
δ_{LIM}	Maximum engine angle	Deg
$\dot{\delta}_{LIM}$	Maximum engine rate	Deg/sec

1. INTRODUCTION

Saturn IB/S-IVB stage separation is possibly the most critical part of S-IVB stage flight. As a result of the recent uprating of the H-1 engine's thrust from 188,000 pounds to 200,000 pounds, and newer aerodynamic data, the previous separation report (reference 1) can no longer be used as an accurate reference.

It is the primary purpose of this report to discuss the results of the controllability analysis made for the Saturn IB configuration using the latest data for thrust buildup curves, aerodynamic coefficients, dynamic pressure, ullage engine configuration, and separation sequencing. The main body of the results are based on "nominal" parameter values which result in small attitude excursions. However, in order to completely cover the range of separation problems some of the more significant parameters such as thrust build up were investigated over a three sigma range. In order to investigate the required aerodynamic dispersions of fifty per cent and one hundred per cent required by reference 2; the dynamic pressure was found to be the most suitable parameter to vary.

2. ANALYSIS METHOD AND PARAMETERS

The major part of this analysis was made by digital simulation of the S-IVB configuration. The dynamic equations used in the simulation include rigid body dynamics only. The autopilot included an attitude and rate of change of attitude feedback. The hydraulic system was approximately by a first order servo with rate and position limits. No sloshing effects were included in the rigid body equations. An analog simulation of the identical problem was made to check the results.

The latest available parameter values were used for the updated Saturn IB configuration. Figure 1 is the block diagram which was used for the analysis. Figure 2 is a drawing of the second stage configuration which includes the S-IVB stage, the LEM, and the Service Module. Figure 3 shows the latest J-2 Engine thrust time history taken from reference 3. Also included in figure 3 is the minimum thrust build up curve guaranteed by Rocketdyne/NAA under contract to NASA (reference 4). The ullage motor thrust time history shown in figure 4 was taken from reference 5. Figures 5 and 6 are plots of the Normal Force Coefficient and Length of Aerodynamic Lever Arm Versus Angle of Attack, respectively.

Tables 1 through 3 represent the remainder of the pertinent variable data used. Table 1 gives the Saturn IB/S-IVB Separation sequencing (reference 6). The engine start time (begin J-2 Chillover) was based on obtaining a sufficient clearance between the end of the engine bell and the aft interstage of the Saturn IB.

Table 2 is a list of the dynamic pressure (q) versus time. This table was referenced to the nominal value of 65.5 kg/m^2 for q at separation that was given in reference 5.

Table 3 represents the range of initial conditions for the initial attitude angle (θ_0), attitude rate ($\dot{\theta}_0$), and the initial angle of attack (α_0). The nominal values in this table have been chosen as the 3 sigma values to be used in S-IVB/IB separation studies. The analysis was extended by parameterizing these conditions in order to give a more complete picture of the separation. The Appendix presents a statistical analysis of initial conditions experienced on the S-IV stage at separation for Saturn I flights, and an extrapolation of this data to the S-IVB/IB stage. This analysis shows that the nominal 3 sigma values used in this study for θ_0 and $\dot{\theta}_0$ are slightly conservative.

Also included in Table 3 is a list of the nominal values used for the engine gimbal angle limit, the engine gimbal rate limit (reference 7), and the assumed thrust misalignment error. The thrust misalignment error includes the errors due to thrust vector misalignment, missile center of gravity offset, thrust offset, and gimbal offset.

3. RESULTS

The separation transient response data given in figures 7 through 17 reflect variations in dynamic pressure, thrust buildup characteristics, ullage motor conditions, thrust vector misalignment, and initial attitude conditions. Two types of results are given, actual transient response, and response trend summaries. The nominal initial conditions are indicated in Table 3.

Figure 7 shows attitude excursion versus time from separation for the "nominal" case. This response is compared to a separation transient using the same initial conditions, but with one ullage engine out. The particular ullage motor out is positioned so that a maximum disturbing moment is produced by the remaining motors thereby resulting in an attitude excursion of 2.8 degrees greater than the nominal case. The engine deflection for this attitude excursion is shown in figure 8.

The effect of dynamic pressure on maximum attitude excursion at separation has been plotted in figure 9. Initial attitude rates of one degree per second and zero degrees per second were used. The upper curve, which resulted from the higher attitude rate, shows considerably larger attitude excursions and is more sensitive to a change in initial dynamic pressure than the lower curve. As much as 30 degrees of attitude angle is obtained for a dynamic pressure of twice the

"nominal" value. Although this is a large attitude excursion, the probability that these initial conditions will occur, based on the statistical study in the Appendix, is small.

Figure 10 shows the effect of the separation sequencing on maximum attitude excursion. The thrust buildup curve of figure 3 was used, and the time from separation to engine start was parameterized. It is seen that with nominal dynamic pressure, sequencing changes have almost no effect on attitude excursion. However, the attitude excursion is quite sensitive to coast time for cases which have one hundred per cent excursion on dynamic pressure.

Figure 11 shows attitude excursion versus initial angle of attack for three sets of initial conditions.

Figure 12 is comparable to figure 11 except the maximum attitude excursion is plotted versus initial attitude pitch rate for an initial attitude angle of 1 degree and a parameterized angle of attack.

Figure 13 represents the effects of changing gimbal limits on attitude excursion and on total controllability and for three values of engine gimbal limit rate ($\dot{\delta} = 5, 10, \text{ and } 15 \text{ degrees/sec.}$)

Because the thrust build-up curve has a very significant effect on the attitude excursion experienced during separation, it is necessary to investigate the effects of both of the minimum three sigma and minimum guaranteed thrust build-up curves shown in figure 3. Figure 14 shows the resulting transient response using three different thrust buildup histories. As much as 4 degrees of added attitude excursion results from using the worst case thrust buildup compared to the nominal thrust buildup.

Figure 15 is similar to figure 14, except that one hundred per cent dispersion on dynamic pressure was used for figure 15. While the vehicle remains stable (transient responses rapidly converge) it is evident that the attitude excursion increases as the dynamic pressure increases. It should be noted that an attitude excursion of 50 degrees occurs for the absolute worst case. This worst case consists of a one hundred per cent dispersion on the aerodynamic pressure and minimum guaranteed thrust buildup time history. The gimbal history for the minimum case is shown in figure 16.

Figure 17 is a summary plot of attitude excursion versus initial dynamic pressure for the three different thrust buildup curves.

Figures 18 and 19 show the effect of steady state attitude control errors on the separation transients due to a 2 degree thrust misalignment resulting from an unsymmetrical thrust and a c g offset. Figure 18 shows the resulting transient response caused by both positive (thrust misalignment moment which acts in the same direction as the aerodynamic moment) and negative misalignment errors for the nominal case. These thrust misalignment errors were simulated by equating the engine error to an equivalent moment of the variable thrust times the variable control lever arm.

The positive thrust misalignment which gives an increase in the destabilizing moment causes a slight increase in the initial attitude excursion but converges slightly faster for the nominal case seen in figure 18.

A positive thrust misalignment error of 2 degrees causes an attitude excursion of about 96° for nominal conditions except for a one hundred per cent dispersion on the aerodynamic pressure. See figure 19. While the probability of these

conditions occurring simultaneously is extremely low, the results must not be neglected.

It is obvious that attitude rates of greater than 9 degrees per second are experienced for extreme disturbance cases such as seen in figure 19. Also, this rate is slightly exceeded for nominal conditions except for one hundred per cent dispersion of dynamic pressure. This results in a serious problem due to the fact that attitude rates of 6 degrees per second may trigger an emergency abort, (Emergency Detection System -- EDS) as outlined in MSFC Ground Rules Document I-CO-VB-4-230.

4. CONCLUSIONS AND RECOMMENDATIONS

4.1 Conclusions

- a. Separation controllability of the Saturn IB/IVB Stage is acceptable (15 deg θ taken from figure 14) when considering the initial conditions of $\theta = 1$ deg; $\dot{\theta} = 1$ deg/sec; $\alpha = 4^\circ$; "Slow" 3σ thrust rise; nominal dynamic pressure and aerodynamic normal force coefficient; and no thrust misalignment. Attitude excursions greater than 45 degrees are unacceptable for the present guidance platform.
- b. The separation attitude excursion using the above conditions but with twice the nominal dynamic pressure or aerodynamic normal force and the minimal guaranteed thrust rise characteristic is unacceptable (51 deg θ) with the present platform (See figure 15).
- c. The separation attitude excursion using the nominal conditions in Table 1, but with twice the nominal dynamic pressure or aerodynamic normal force coefficient and 2° thrust misalignment is also unacceptable (96 deg θ) with the present platform (See figure 19).

- d. The probability that the "nominal" initial conditions for the attitude rate and angle will occur is unlikely.
- e. Small deviations (from 1.25 seconds to 1.84 seconds) in the time from separation at which the engine start signal is initiated causes less than 2 degrees of additional deviation in the attitude excursion (See figure 10) for a system experiencing "nominal" initial conditions.

4.2 Recommendations

- a. For a simultaneous application of several 3 sigma disturbances and variations, unacceptable attitude deviations are experienced. Additional analyses should be conducted to establish probabilities to these excursions and recommend design changes if necessary (See figure 19).
- b. Further analysis will be required to determine a value for the attitude rate to be used as part of the emergency detection system.

5. BIBLIOGRAPHY

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6. MSFC Memorandum R-P&VE-VOI-64-16, "SA-201 Flight Sequence," dated May 5, 1964.
7. Douglas Report DS-2163, "Model Specification Saturn S-IVB Stage," dated February, 1963.
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9. "The Elements of Probability Theory," Harold Cramer, John Wiley and Sons, New York, dated October, 1955.
10. Douglas Report SM-46569, "An Engine Out Controllability Study of the S-IB Stage of the Saturn IB Vehicle Model DSV-4B, dated April, 1964.
11. MSFC Ground Rules Document I-CO-VB-4-230, dated August 20, 1964.

TABLE 1

S-IB/S-IVB SEPARATION SEQUENCE

T - 6.4 sec.	Cutoff inboard engines
T - 0.4 sec.	Cutoff outboard engines
T - 0.1 sec.	Ignite ullage motors
T	Initiate separation cutting and retro motors
T + 0.02 sec.	Cutting complete
T + 0.03 sec.	Retromotor thrust buildup begins
T + 0.20 sec.	S-IVB roll control system activated
T + 1.60 sec.	Begin J-2 chilldown (engine start signal)
T + 2.70 sec.	J-2 Thrust Buildup Begins (Initiate engine gimballing)
T + 4.92 sec.	J-2 engine at 90% thrust

TABLE 2

DYNAMIC PRESSURE VERSUS TIME FROM IGNITION

TIME	DYNAMIC PRESSURE	
Seconds	lb/ft ²	Kg/m ²
-6	23.8	114.0
-2	16.55	79.5
0	13.7	65.6
2	10.6	51.0
4	8.15	39.1
6	6.20	29.8
8	4.75	22.8
18	1.20	5.77
25	0	0

TABLE 3

RANGE OF INITIAL CONDITIONS

		NOMINAL VALUE
Initial Dynamic Pressure, Q_0	65.6 Kg/m ² to 131.2 Kg/m ²	65.6
Initial Angle of Attack α_0	-1° to +4°	+4°
*Initial Attitude Angle, θ_0	-1° to +1°	+1°
*Initial Attitude Rate $\dot{\theta}_0$	-1°/sec to +1°/sec	+1°/sec
Thrust Vector Misalignment (θ_0)	-2° to +2°	0°
Time of Zero Dynamic Pressure		25 sec

Nominal Engine Parameters

Engine Gimbal Angle Limit $\theta_{Lim} = 7^\circ$ Engine Gimbal Rate Limit $\dot{\theta}_{Lim} = 8^\circ/\text{sec}$

*Note: See appendix for discussion of initial values of θ and $\dot{\theta}$ using the previous Block 1, Saturn 1 flights to determine statistical values for θ_0 and $\dot{\theta}_0$.

[illegible]

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SATURN 1B SECOND STAGE CONFIGURATION

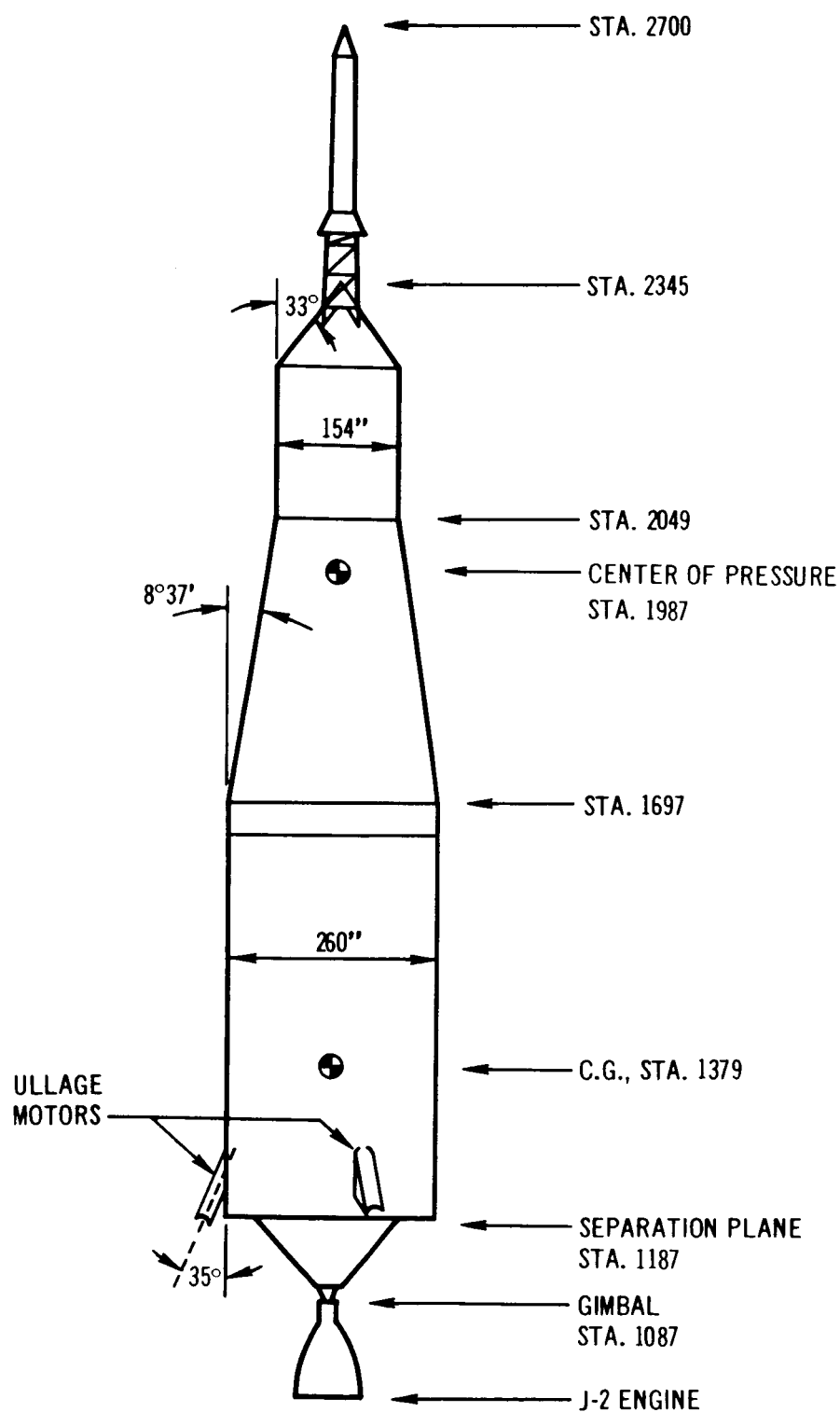


FIGURE 2

PREDICTED J-2 ENGINE THRUST BUILDUP TIME HISTORY

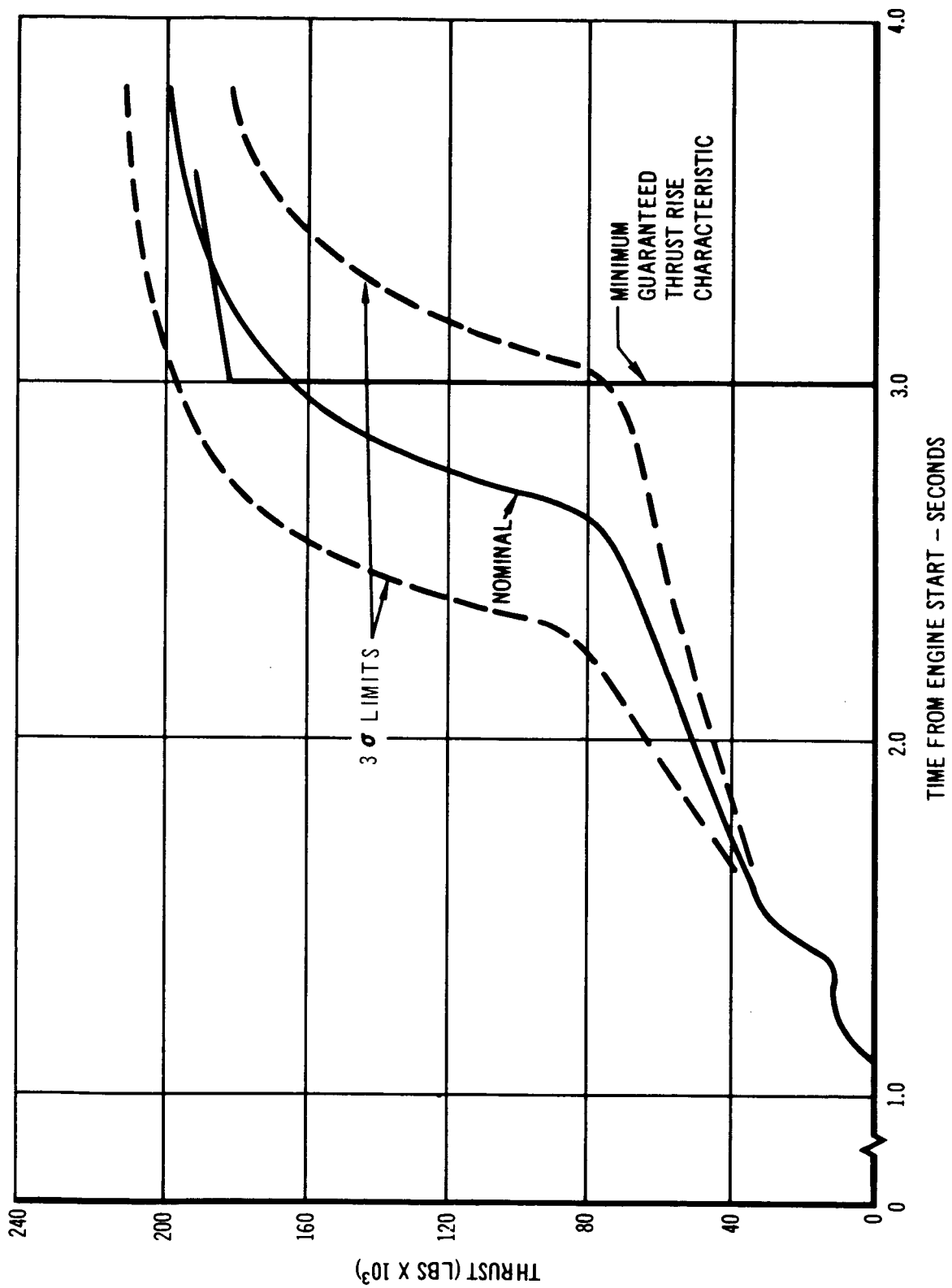


FIGURE 3

ULLAGE MOTOR THRUST TIME HISTORY

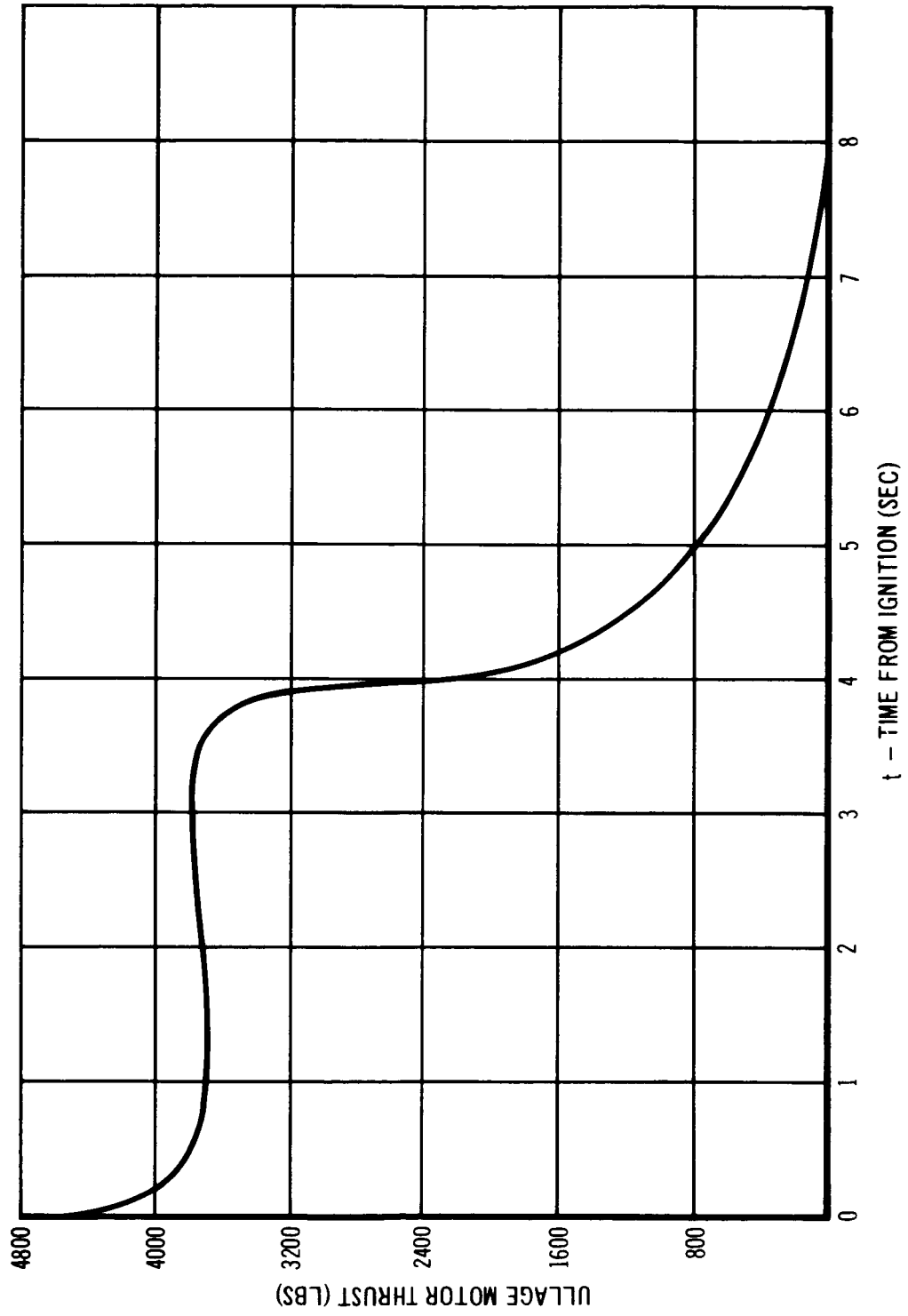


FIGURE 4

NORMAL FORCE COEFFICIENT VS. ANGLE OF ATTACK

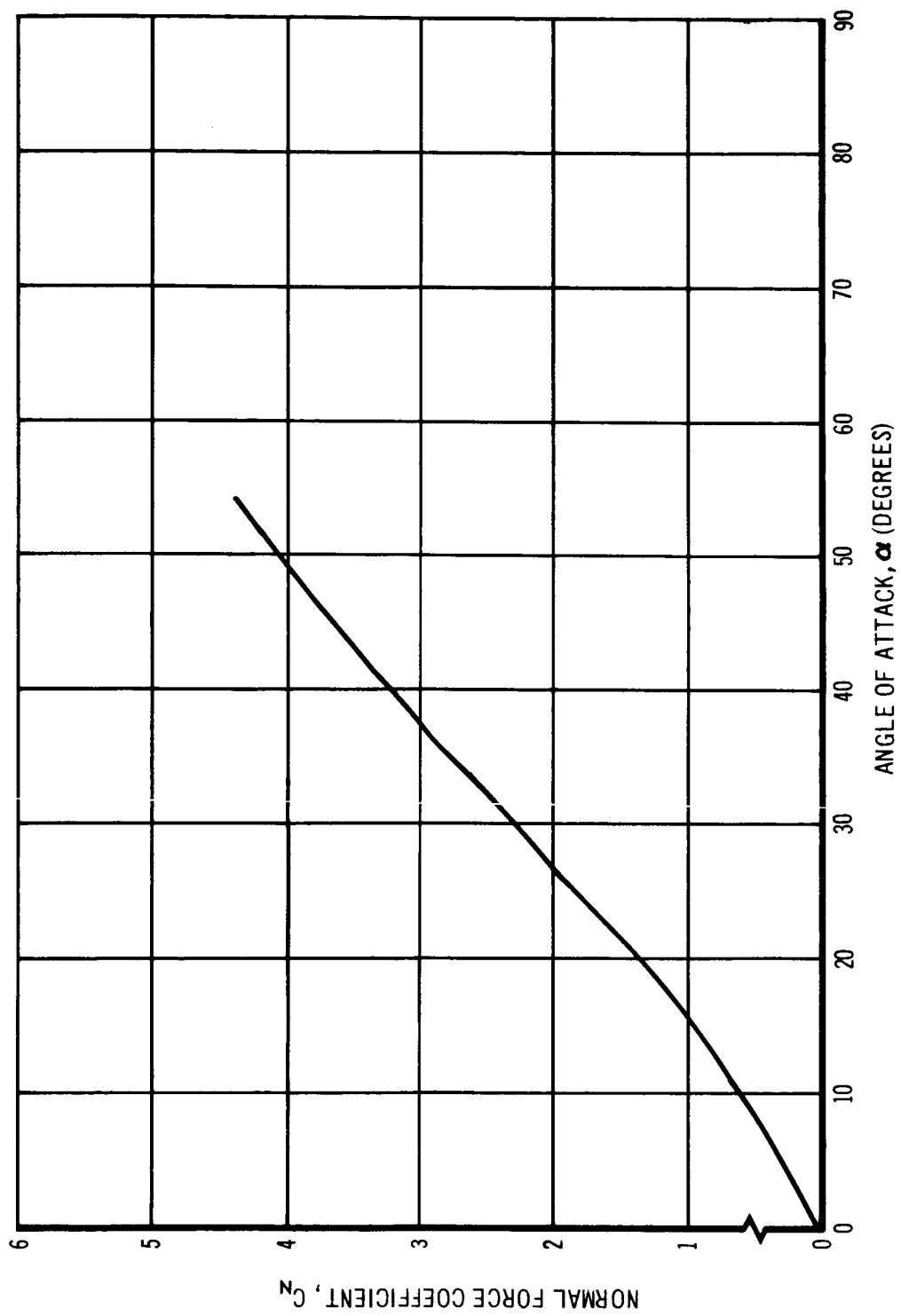


FIGURE 5

LENGTH OF AERODYNAMIC LEVER ARM VS. ANGLE OF ATTACK

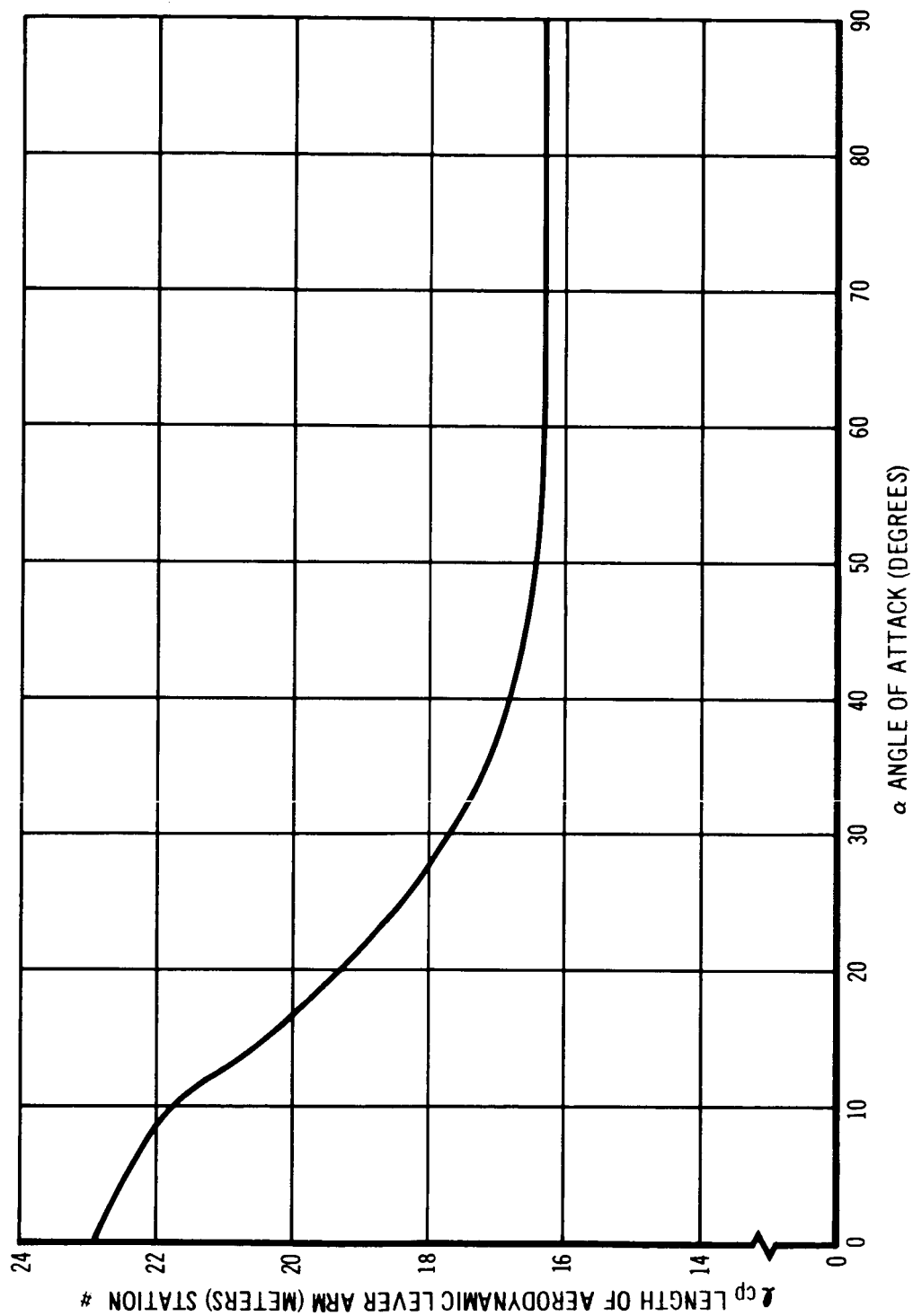


FIGURE 6

ATTITUDE ANGLE VERSUS TIME FROM SEPARATION FOR NOMINAL CASE
WITH AND WITHOUT ONE ULLAGE MOTOR OUT

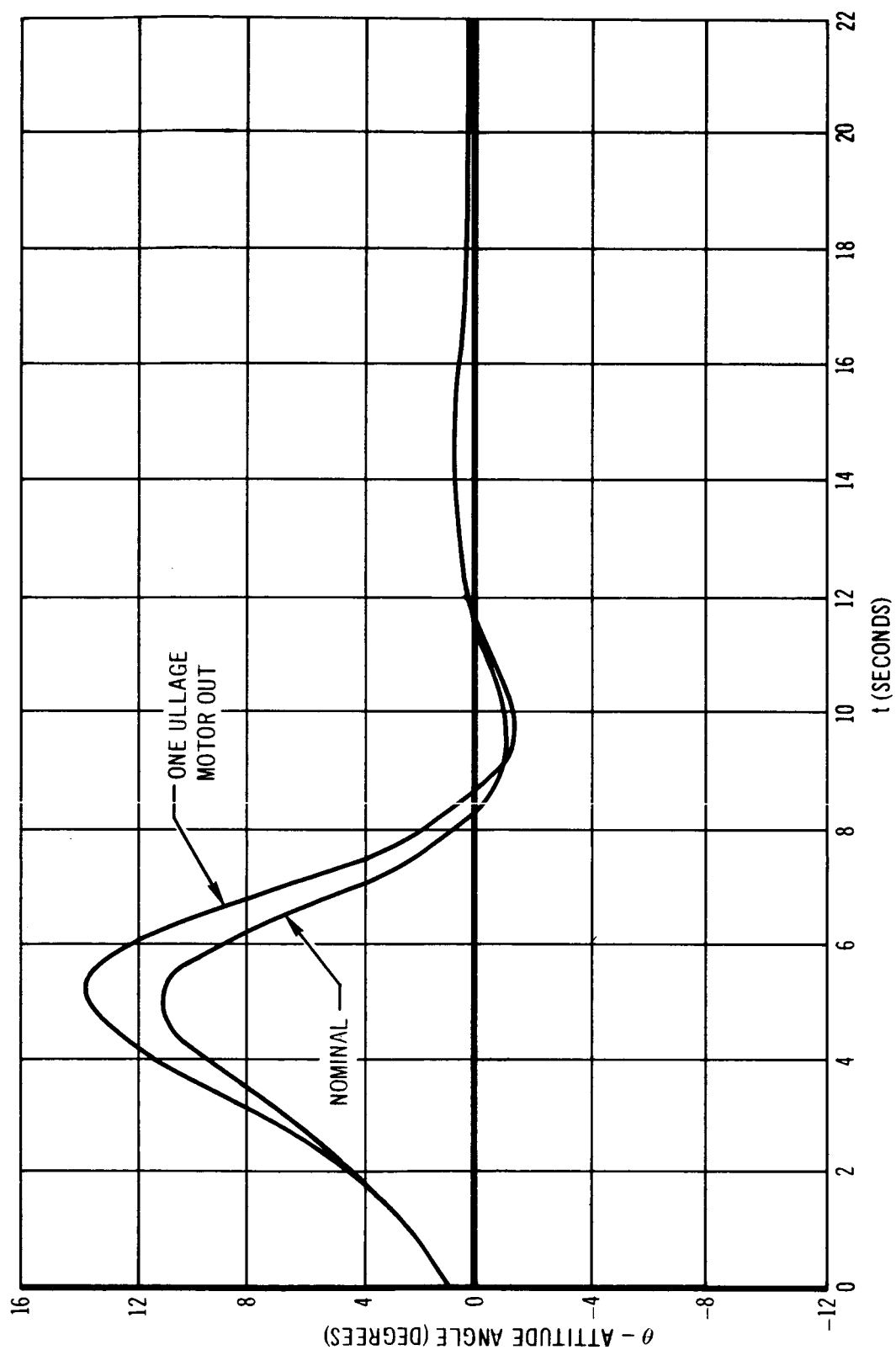


FIGURE 7

ENGINE GIMBAL HISTORY FOR NOMINAL CASE

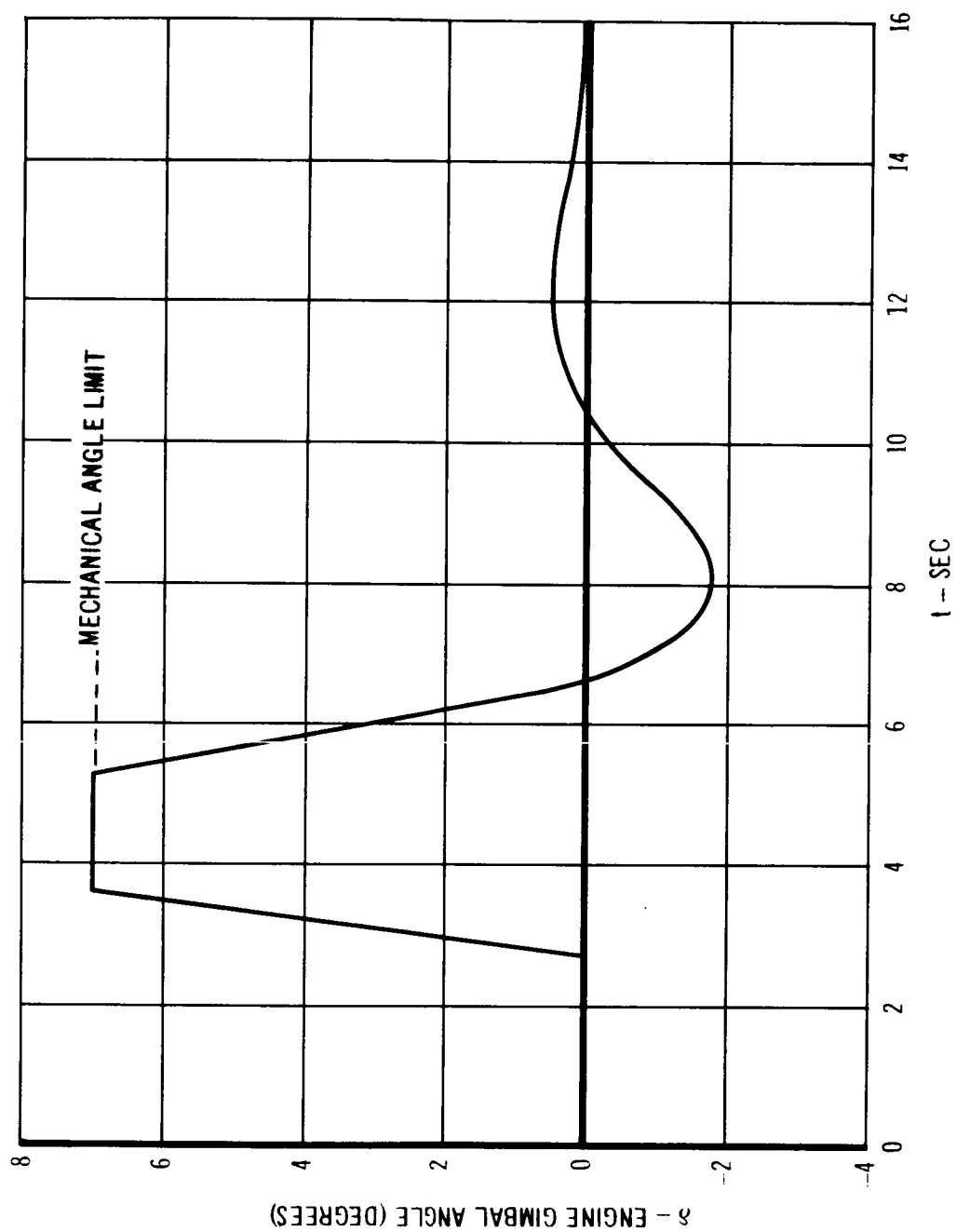


FIGURE 8

MAXIMUM ATTITUDE EXCURSION VERSUS INITIAL DYNAMIC PRESSURE
FOR NOMINAL CASE AND FOR REDUCED INITIAL ATTITUDE RATE

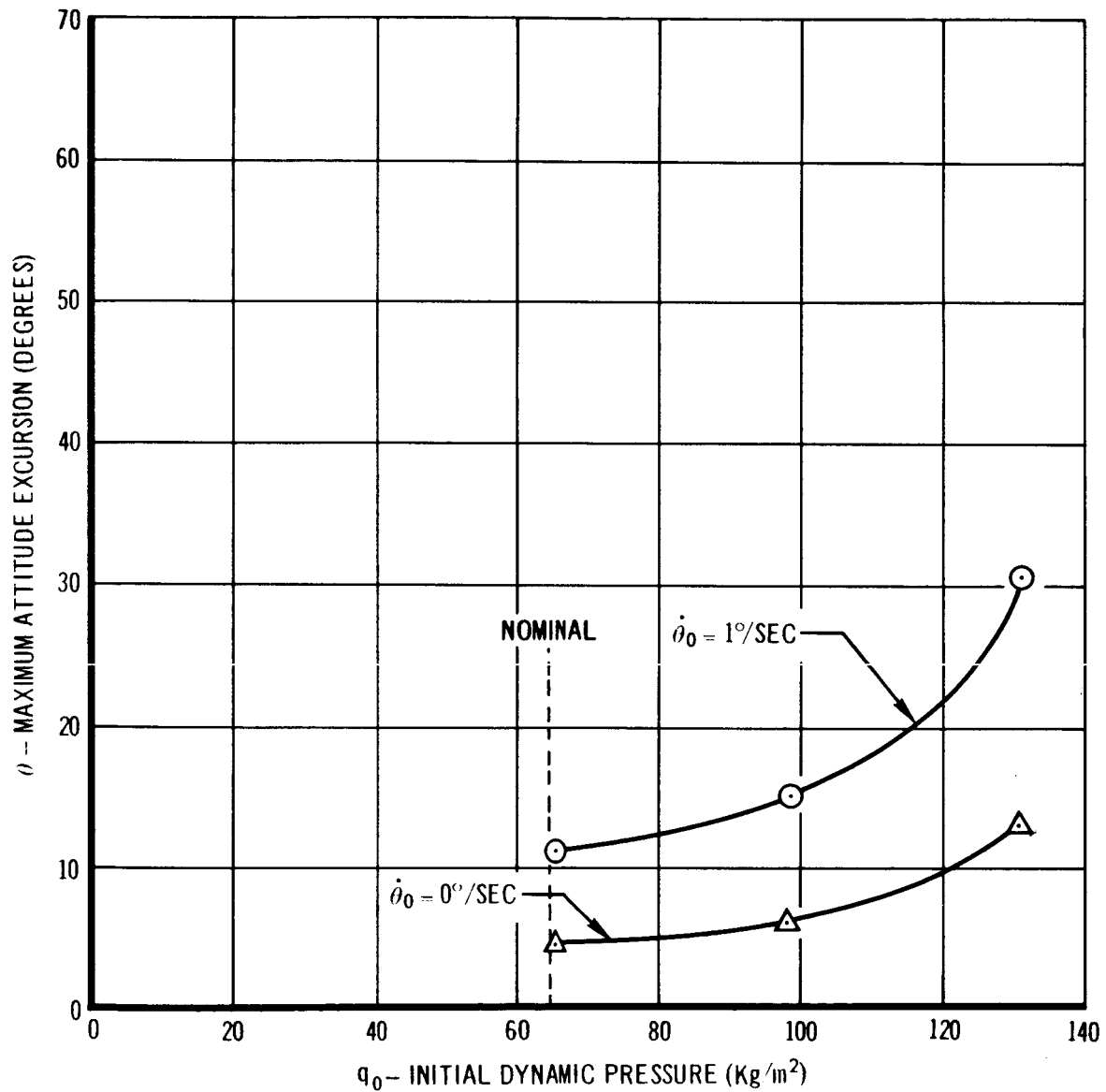


FIGURE 9

MAXIMUM ATTITUDE EXCURSION VERSUS TIME FROM SEPARATION TO J-2 ENGINE
START FOR NOMINAL CONDITIONS AND VARYING DYNAMIC PRESSURE

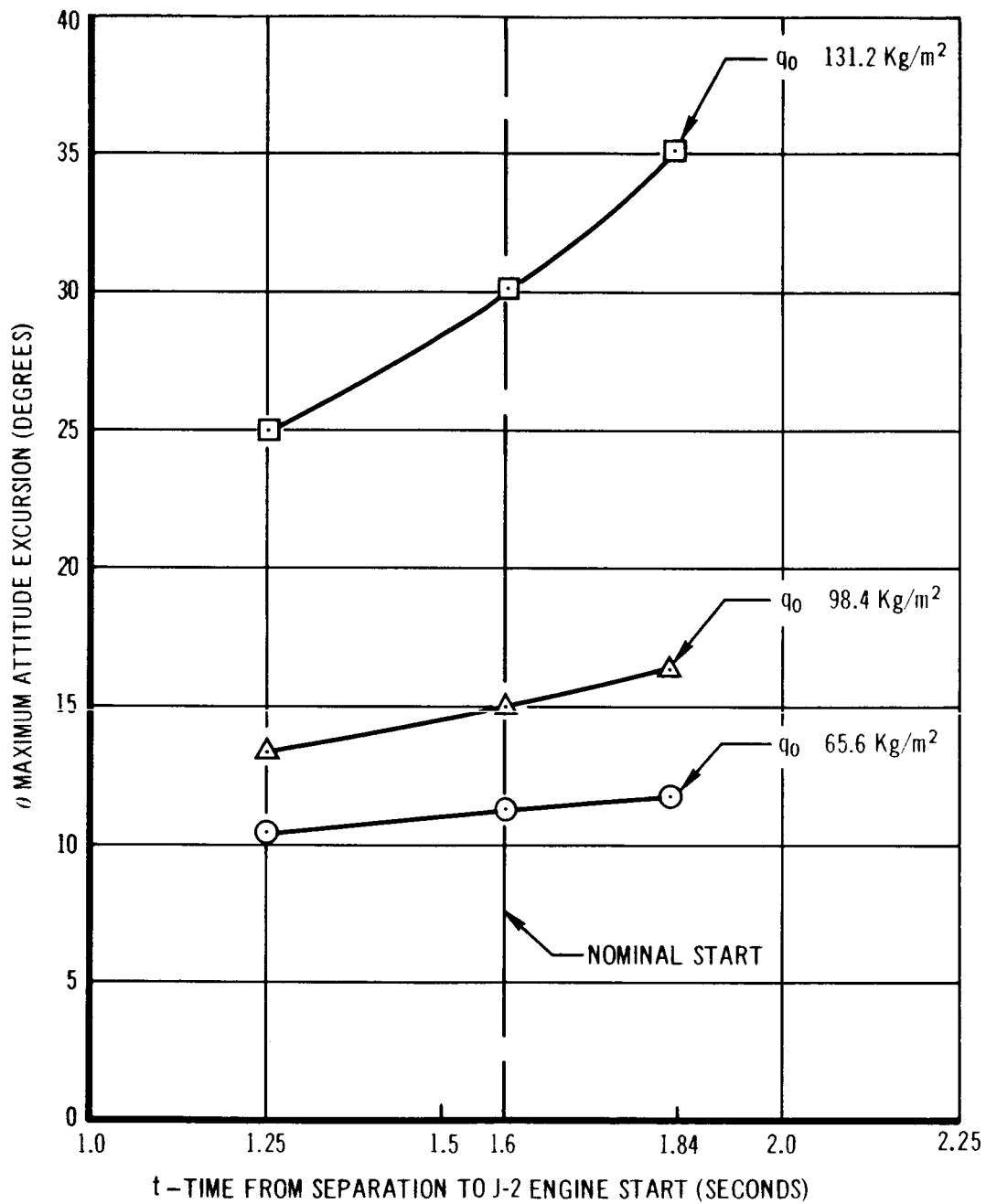


FIGURE 10

MAXIMUM ATTITUDE EXCURSION VERSUS INITIAL ANGLE OF ATTACK FOR NOMINAL CONDITIONS

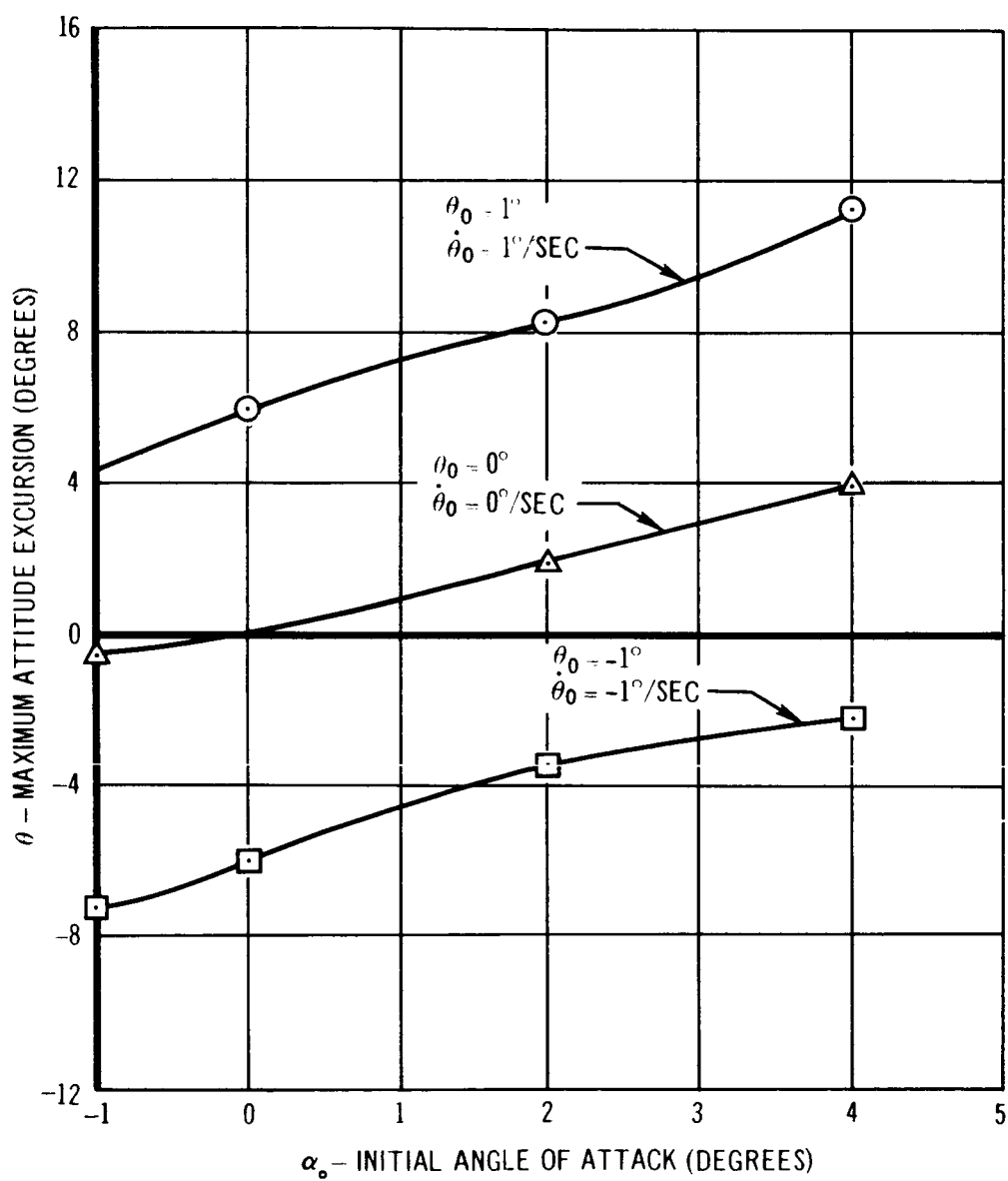


FIGURE 11

MAXIMUM ATTITUDE EXCURSION VERSUS INITIAL PITCH RATE
FOR NOMINAL CONDITIONS EXCEPT AS NOTED

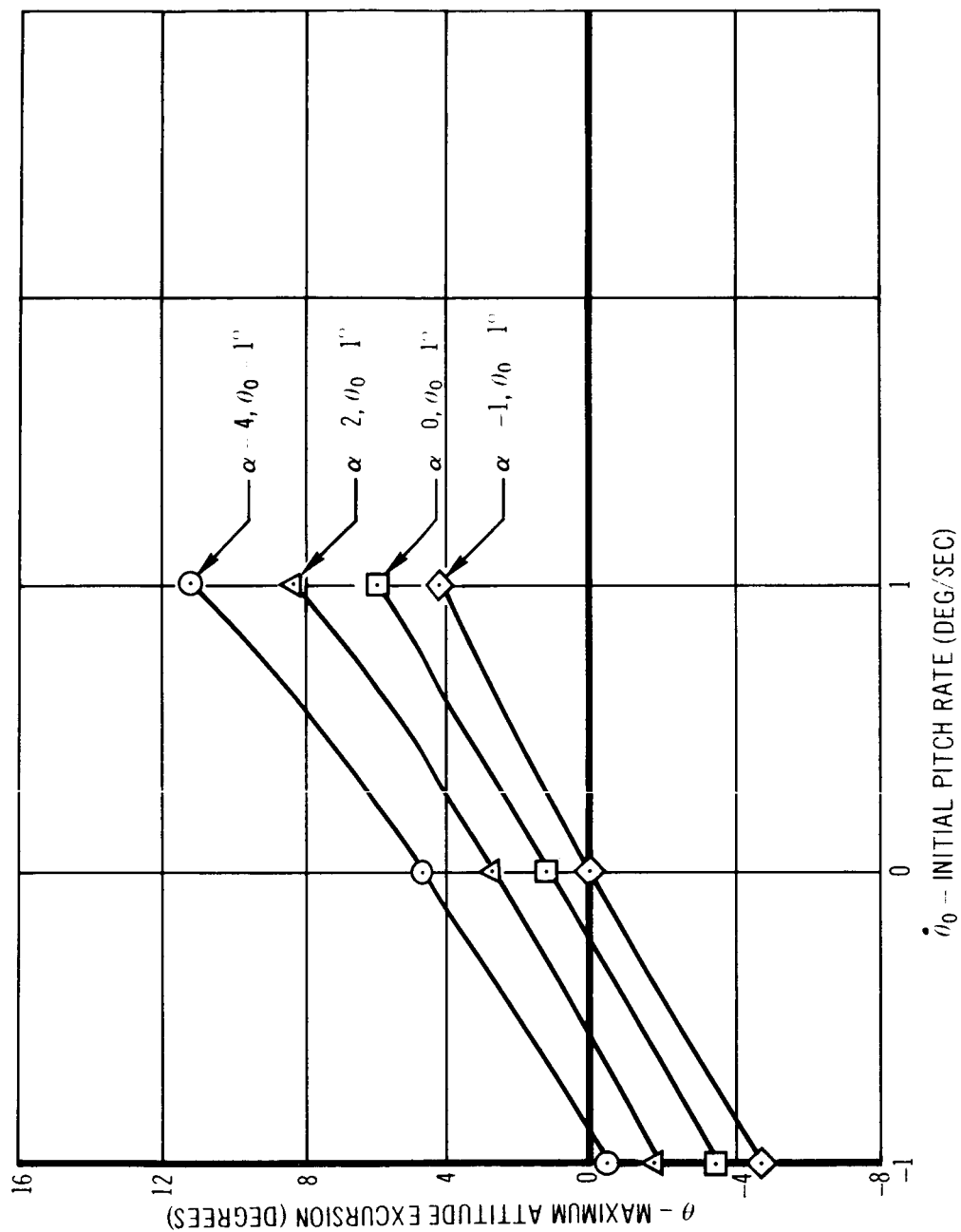


FIGURE 12

MAXIMUM ATTITUDE EXCURSION VERSUS ENGINE GIMBAL ANGLE LIMIT FOR
VARIOUS ENGINE GIMBAL RATE LIMITS USING NOMINAL INITIAL CONDITIONS

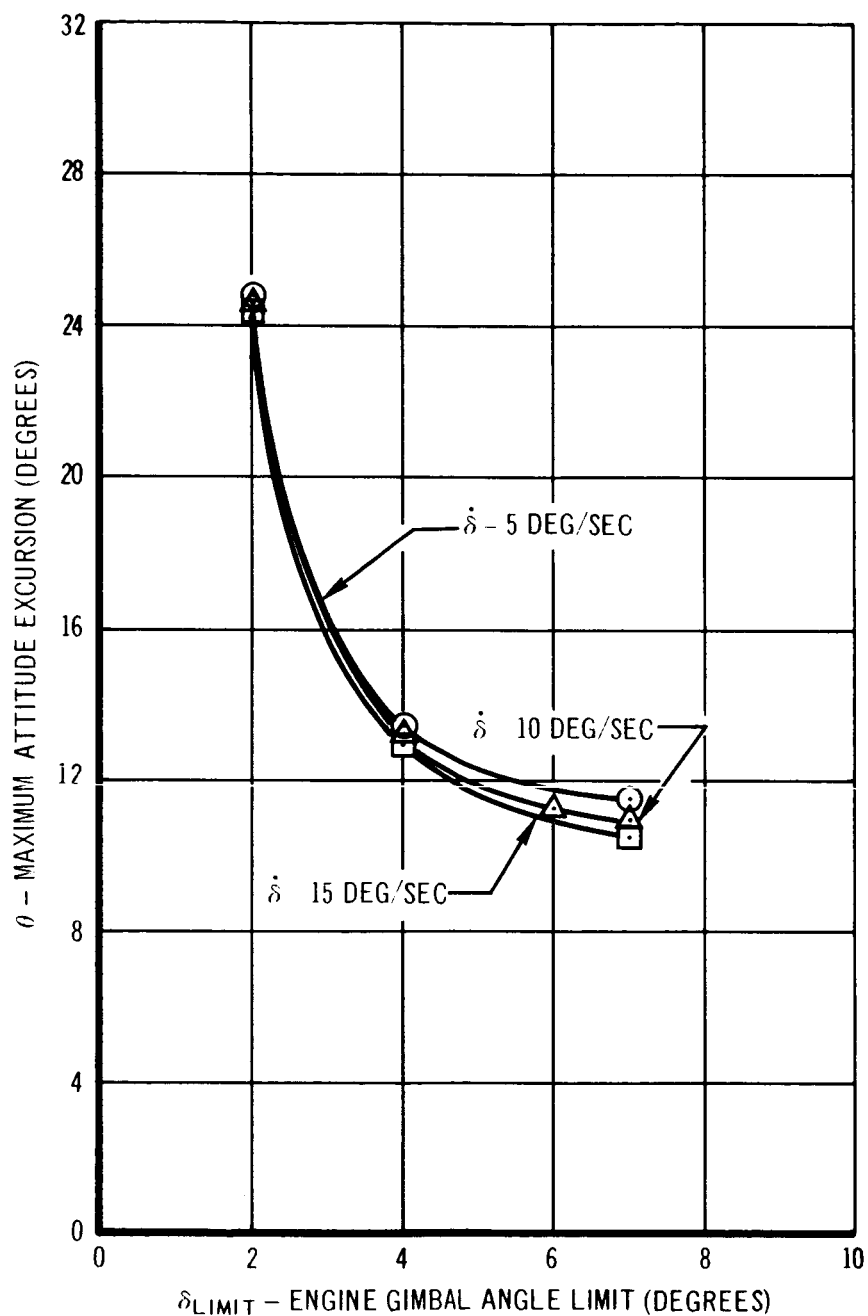


FIGURE 13

ATTITUDE ANGLE VERSUS TIME FROM SEPARATION FOR NOMINAL CONDITIONS
AND THREE DIFFERENT THRUST BUILDUP CURVES

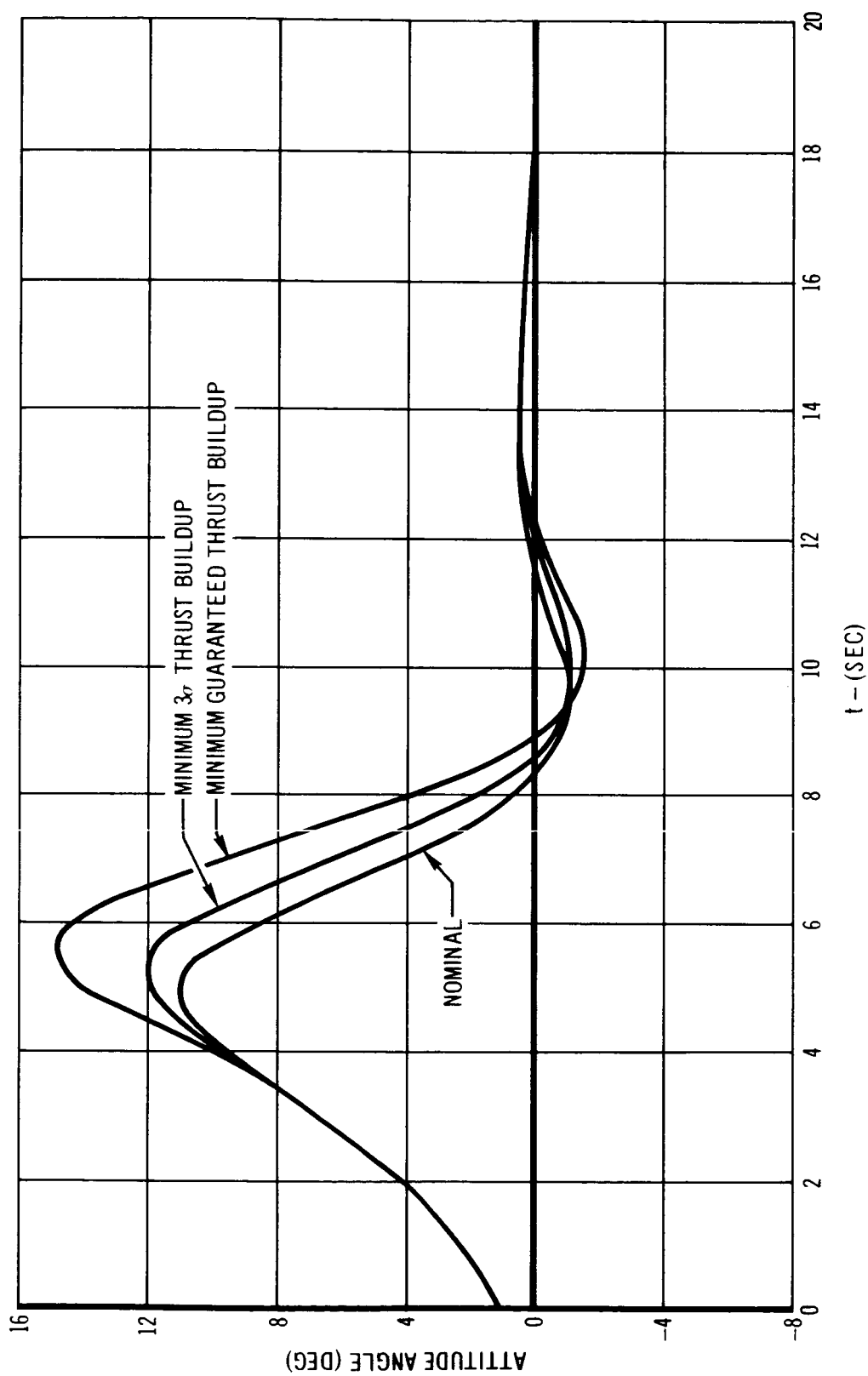


FIGURE 14

ATTITUDE ANGLE VERSUS TIME FROM SEPARATION FOR NOMINAL CONDITIONS
EXCEPT FOR THREE DIFFERENT THRUST BUILD UP CURVES AND A ONE
HUNDRED PERCENT DISPERSION ON DYNAMIC PRESSURE

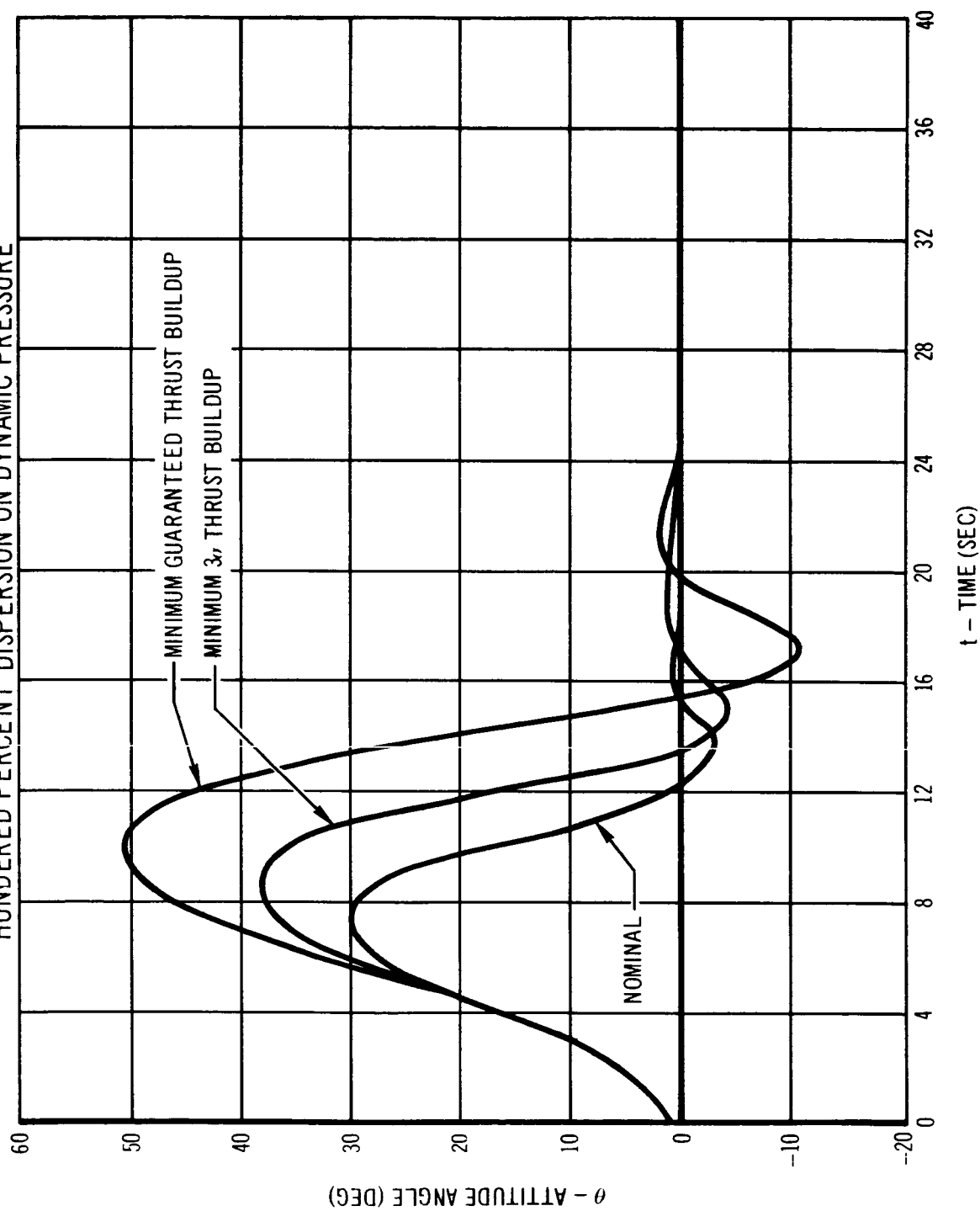


FIGURE 15

ENGINE GIMBAL HISTORY FOR NOMINAL CASE EXCEPT FOR A ONE HUNDRED
PERCENT DISPERSION ON THE AERODYNAMIC PRESSURE

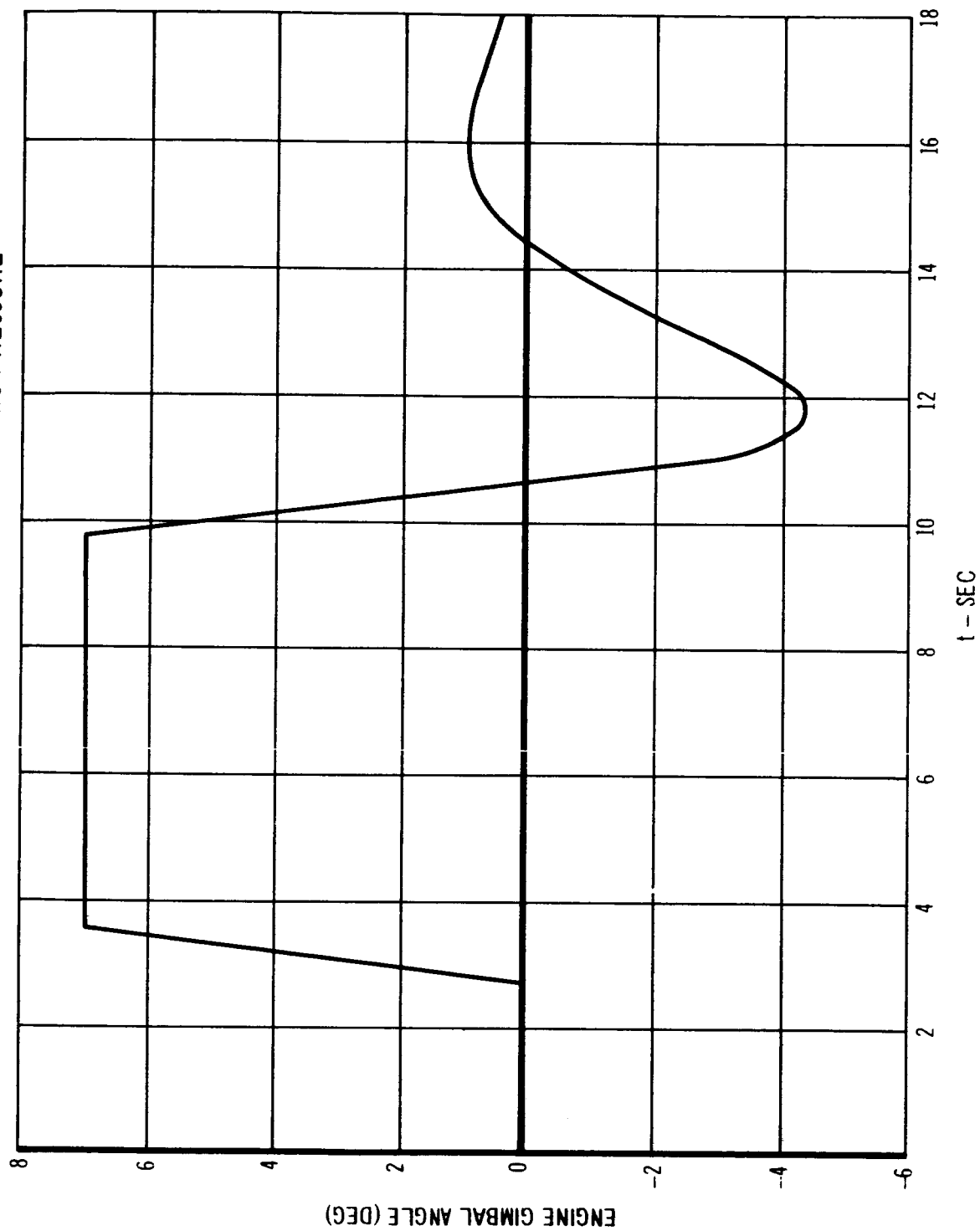


FIGURE 16

ATTITUDE EXCURSION VERSUS INITIAL DYNAMIC PRESSURE FOR NOMINAL
CONDITIONS EXCEPT FOR DIFFERENT THRUST RISE CURVES

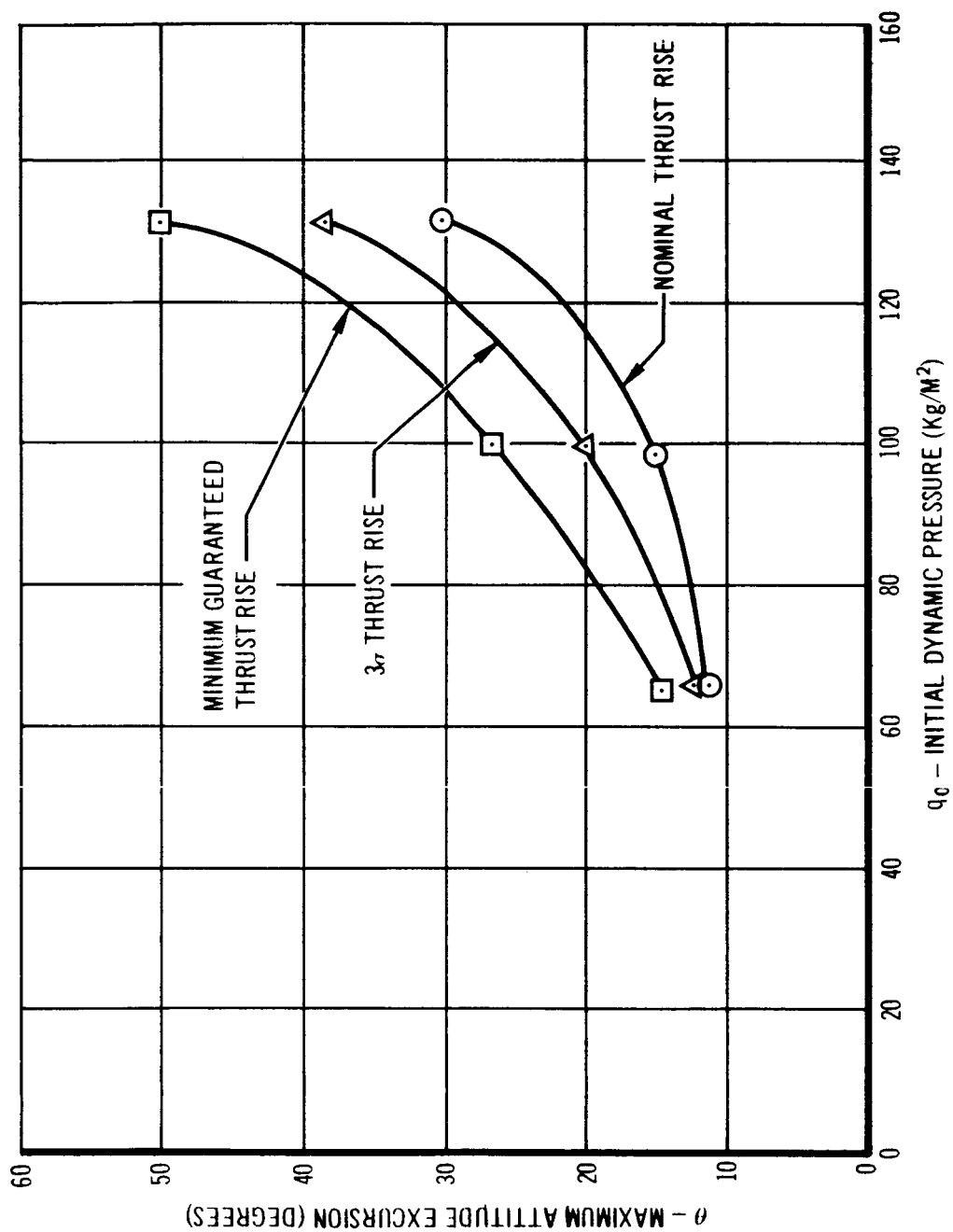


FIGURE 17

ATTITUDE ANGLE VERSUS TIME FROM SEPARATION SHOWING THE EFFECT
OF A THRUST MISALIGNMENT ON THE TRANSIENT RESPONSE FOR
NOMINAL CASE

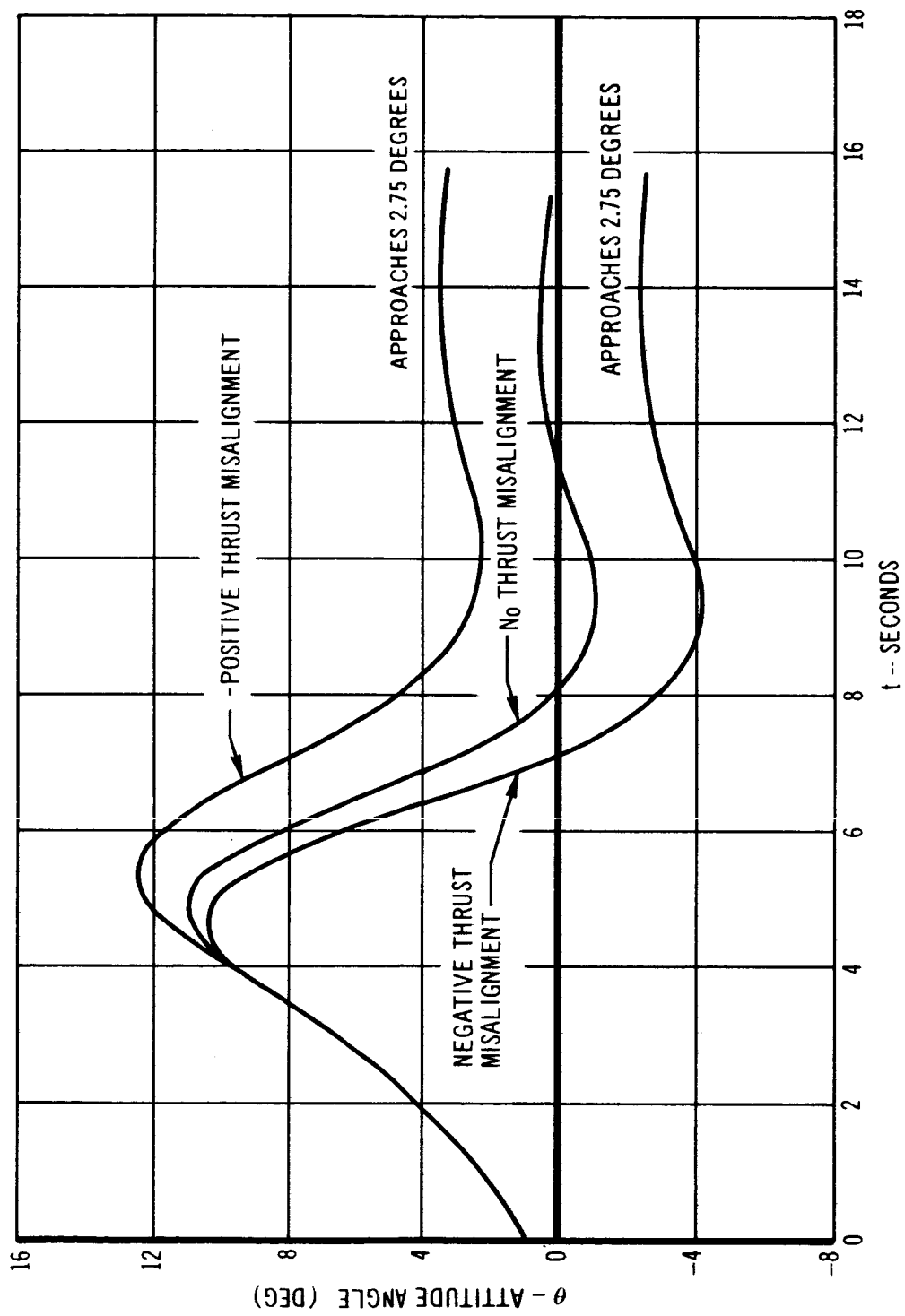


FIGURE 18

ATTITUDE ANGLE VERSUS TIME FROM SEPARATION SHOWING THE EFFECT
OF THRUST MISALIGNMENT ON THE TRANSIENT RESPONSE FOR A
NOMINAL CASE EXCEPT FOR A ONE HUNDRED PERCENT DISPERSION
ON AERODYNAMIC PRESSURE

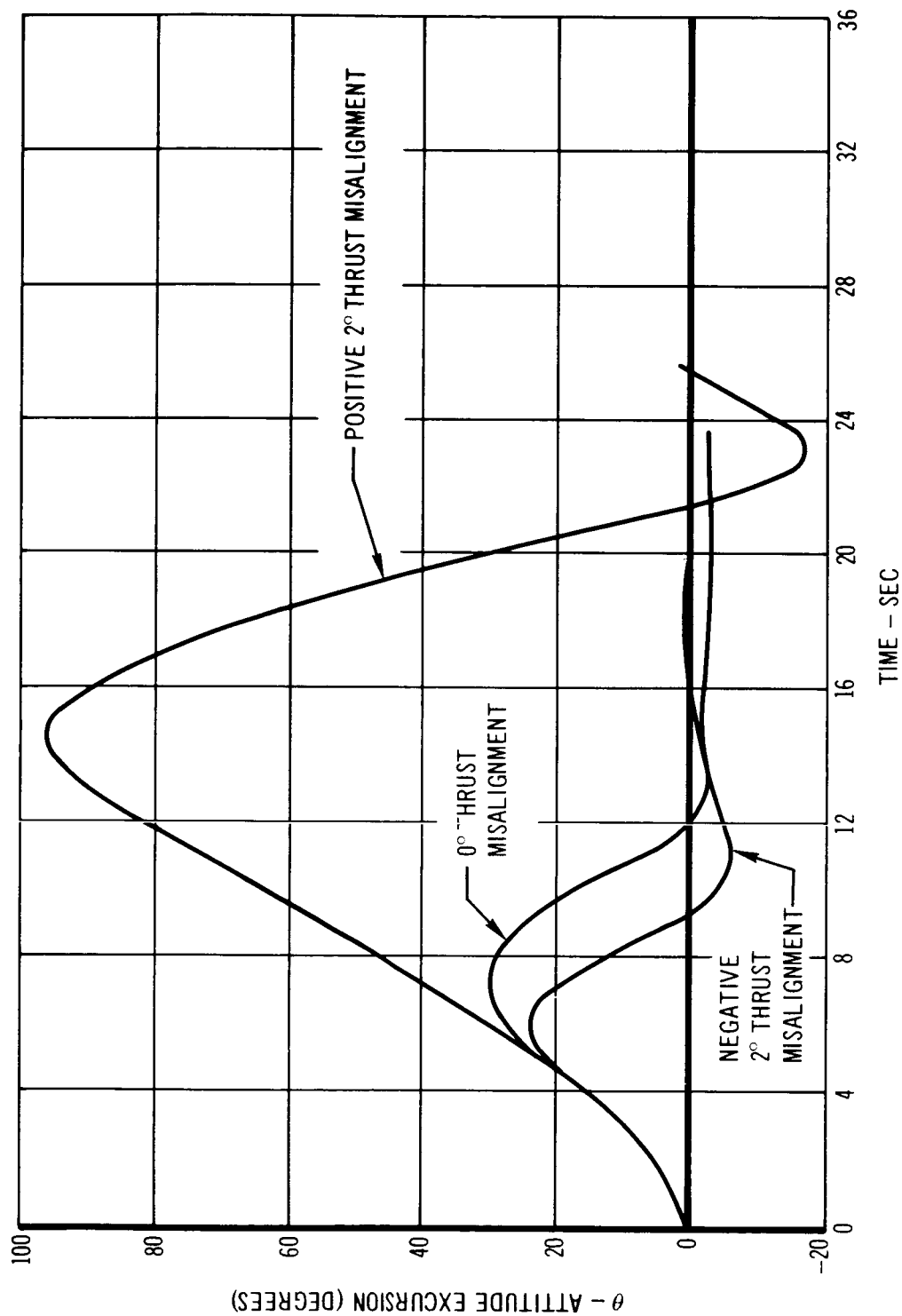


FIGURE 19

APPENDIX

APPENDIX

STATISTICAL ANALYSIS OF THE SATURN IB/S-IVB STAGE SEPARATION ON THE INITIAL CONDITIONS FOR PITCH ANGLE AND PITCH RATE

INTRODUCTION

This appendix has been written in order to compare the range of final first stage conditions which occurred on the first five Saturn I flights with the initial S-IVB conditions used in this report. The actual initial conditions were transformed into statistical deviations which are a function of statistical confidence limits. It is realized that the Saturn I may result in different conditions of separation due to its lower thrust capability (the H-1 engines will be uprated from 188K to 200K pounds of thrust for the Saturn IB flights) different inertia, and different aerodynamic characteristics. Consequently, a method was developed to extrapolate Saturn I data to Saturn IB.

The method of analysis consisted of using the final conditions from the first five Saturn I flights. Using these values a statistical confidence interval of 90 per cent was chosen arbitrarily for determining the three sigma interval for final conditions of the Saturn IB first stage, (equivalent to the initial conditions for initial attitude angle θ_0 and initial attitude rate $\dot{\theta}_0$ of the S-IVB second stage). It should be noted that several assumptions were made in order to carry out this analysis. These assumptions include the following:

- a. The pitch and yaw axes of the vehicle are symmetrical thereby making it possible to increase the sample size from five to ten by including both the pitch and yaw values.
- b. The values were assumed to be distributed such that the actual mean value for θ_0 and $\dot{\theta}_0$ was zero.
- c. A confidence interval of 90 per cent was chosen because the sample size of 10 was considered to be very small.

PROCEDURE AND CALCULATIONS

The following ten samples are given for both θ_o and $\dot{\theta}_o$ where the pitch and yaw samples are orthogonal and therefore considered independent.

(Sample No.)	$\left (\theta_o)\right $ (Deg.)	$\left (\dot{\theta}_o)\right $ (Deg/Sec)
1	0.08	0.01
2	0.06	0.03
3	0.03	0.21
4	0.05	0.29
5	0.07	0.09
6	0.20	0.05
7	0.05	0.04
8	0.09	0.10
9	0.11	0.10
10	0.45	0.09

The preceding samples were taken from the flight evaluation results given by reference 8 where polarity was not specified. As discussed above, it was assumed that the means of θ_o and $\dot{\theta}_o$ were both zero.

Using the equations found in reference 9 for determining the actual range for the variance corresponding to a given confidence limit we have:

$$P \left\{ \chi_{p'}^2 < \frac{ns^2}{\sigma^2} < \chi_{p''}^2 \right\} = 1 - \frac{P}{100}$$

or

$$P \left\{ \frac{s\sqrt{n}}{\chi_{p''}} < \sigma < \frac{s\sqrt{n}}{\chi_{p'}} \right\} = 1 - \frac{P}{100}$$

where:

S - Standard deviation for the sample

σ - Statistical standard deviation

n - Sample size

p - Level of significance

χ^2 - Chi Squared Distribution

and

$$p' = 100 - 1/2p$$

$$p'' = 1/2 p$$

and

$$s = \sqrt{\frac{1}{n} \sum_{L=1}^n (\bar{X} - x_L)^2}$$

where

\bar{X} = mean value of samples

x_L = individual samples

First looking at θ we have

$$\bar{X} = \frac{\sum x_L}{n} = 0$$

$$s_{\theta} = \sqrt{\frac{1}{n} \sum (\bar{X} - x_L)^2} = \sqrt{\frac{0.2835}{10}} = \sqrt{0.02835} = 0.168$$

For 90 per cent confidence $p = 10$

then

$$p' = 100 - (1/2)p = 100 - 5 = 95$$

$$p'' = (1/2)p = 5$$

Using the tables for Chi Square Distribution

$$\chi_{p'}^2 = 3.940 \quad \text{or} \quad \chi_{p'} = 1.99$$

$$\chi_{p''}^2 = 18.307$$

$$P \left[\frac{s_{\theta} \sqrt{n}}{\chi_{p''}} < \sigma < \frac{s_{\theta} \sqrt{n}}{\chi_{p'}} \right] \quad \text{we are primarily interested in the maximum limit.}$$

therefore

$$\sigma < \frac{s_{\theta} \sqrt{n}}{\chi_{p'}} = 0.268$$

or

$$3_{\sigma} = 0.804 \text{ degrees for } \dot{\theta}$$

Using similar methods of calculations and a 90 per cent confidence interval the following results were obtained for $\dot{\theta}_0$.

$$\bar{X}_{\dot{\theta}} = 0 \text{ degrees/sec} \quad (\text{Assumption No. 2})$$

$$S_{\dot{\theta}} = 0.130 \text{ degrees/sec}$$

$$\sigma_{\dot{\theta}} = 0.208 \text{ degrees}$$

The three sigma value for the initial pitch rate is 0.624 degrees/second with 90 per cent confidence.

From the results of these calculations, for a 90 per cent confidence interval, it is seen that on the Saturn I-S-IV stage the three sigma value for the initial attitude angle (θ_0) was 0.804 degrees and the three sigma value for the initial pitch rate ($\dot{\theta}_0$) also for 90 per cent confidence was 0.624 degrees per second. In other words there is only a ten per cent chance of a value falling outside the confidence interval or a five per cent change of a value falling outside the maximum side of the interval.

It should also be pointed out that because the assumption of lumping both the pitch and yaw values together was made; both the pitch and yaw axis will experience the same initial condition of the above determined values. The major reason for attacking the problem in this manner is due to the extreme sensitivity on the confidence interval for different sample sizes.

In order to show some correlation between the end conditions of the first five Saturn I flights and the forthcoming Saturn IB flights, the accelerations caused by aerodynamic moments and unsymmetrical thrust decay were compared.

A comparison of Saturn I and Saturn IB aerodynamic accelerations is:

Saturn IB

$$\frac{qC_{z\alpha}l_{cp}}{I} = 0.12 \times 10^{-2} \text{ rad/sec}^2/\text{rad}$$

$$\frac{T}{I} = 6.24 \times 10^{-2}/\text{ft-sec}^2$$

Saturn I

$$\frac{qC_{z\alpha}l_{cp}}{I} = 0.226 \times 10^{-2} \text{ rad/sec}^2/\text{rad}$$

$$\frac{T}{I} = 8.37 \times 10^{-2}/\text{ft-sec}^2$$

As a result of the lower T/I ratio for the Saturn IB, unsymmetrical thrust tail-off will produce lower vehicle accelerations and thus lower attitude errors and attitude rates at separation than on Saturn I (reference 10.) The lower acceleration due to aerodynamic force for the Saturn IB vehicle will also result in lower attitude disturbances at separation from winds or angle of attack errors. Thus, it appears that the statistical range of standard deviations for initial attitude angle and rate used for S-IB/S-IVB separation are conservative.